Unit 1: Algebra & Number (Part A)

Part A consists of 3 outcomes that need to be covered:

✓ **AN1: Demonstrate an understanding of factors of whole numbers** by determining the prime factors; greatest common factor; least common multiple; square root; and cube root.

✓ **AN2: Demonstrate an understanding of irrational numbers** by representing, identifying, simplifying and ordering irrational numbers.

AN3: Demonstrate an understanding of powers with integral and rational components.

Outcome AN3 will be covered in Sections 4.4, 4.5, and 4.6. Today, we will look at the following:

Express powers with rational exponents as radicals and vice versa, when m and n are natural numbers, and x is a rational number,

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$ and $\left(x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}\right) = \sqrt[n]{x^m}$

Activate Prior Learning:

Exponent Laws

Product of powers law: $a^m \cdot a^n = a^{m+n}$

$x^2 \cdot x^3 = x^5$

Quotient of powers law: $\dfrac{a^m}{a^n} = a^{m-n}$

$\dfrac{x^5}{x^2} = x^3$

Power of a power law: $(a^m)^n = a^{mn}$

$(x^2)^3$

x^6

Write as a single power.

a) $3^2 \cdot 3^5$

b) $(4^2)^5$

c) $(-5)^{10} \div (-5)^8$

4.4 Fractional Exponents and Radicals

LESSON FOCUS Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?

Use a calculator to complete the table.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$ 1
4	$4^{\frac{1}{2}} =$ 2
9	$9^{1/2} = 3$
16	$16^{1/2} = 4$
25	$25^{1/2} = 5$

4.4 Fractional Exponents and Radicals

Use a calculator to complete the table.

x	$x^{\frac{1}{3}}$
1	
8	$8^{1/3} = 2$
27	$27^{1/3} = 3$
64	
125	

$\frac{1}{3} = 0.\bar{3}$

0.333333

4.4 Fractional Exponents and Radicals

$a^{1/2} = \sqrt{a}$

$a^{1/3} = \sqrt[3]{a}$

$a^{2/3} = \left(\sqrt[3]{a}\right)^2$ index

Example #1

$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

#2 $\left(\sqrt[5]{70}\right)^3 = 70^{3/5}$

$\left(\sqrt[6]{666}\right)^6$

$= 666^{6/6} = 666^1 = 666$

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{27}{8}}\right)^2$$

$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$$

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} \quad \text{and} \quad \sqrt{5} \cdot \sqrt{5} = \sqrt{25}$$

$$= 5^1 \qquad\qquad = 5$$

$$= 5$$

$$\sqrt[3]{\left(\frac{3}{8}\right)^4} = \left(\frac{3}{8}\right)^{4/3}$$

numerator

denominator

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} =$$

$$= \frac{9}{4} = 2\frac{1}{4}$$

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

SOLUTION

CHECK YOUR UNDERSTANDING

$$40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2$$

$$40^{2 \cdot \frac{1}{3}}$$

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

SOLUTION

a) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

$$40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2 \text{ or } \sqrt[3]{40^2}$$

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

SOLUTION

The denominator of the exponent is the index of the radical.

a) $$\begin{aligned} 27^{\frac{1}{3}} &= \sqrt[3]{27} \\ &= 3 \end{aligned}$$

b) $$\begin{aligned} 0.49^{\frac{1}{2}} &= \sqrt{0.49} \\ &= 0.7 \end{aligned}$$

c) $$\begin{aligned} (-64)^{\frac{1}{3}} &= \sqrt[3]{-64} \\ &= -4 \end{aligned}$$

d) $$\begin{aligned} \left(\frac{4}{9}\right)^{\frac{1}{2}} &= \sqrt{\frac{4}{9}} \\ &= \frac{2}{3} \end{aligned}$$

CHECK YOUR UNDERSTANDING

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

SOLUTION

a) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

$$40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2 \text{ or } \sqrt[3]{40^2}$$

b) Use $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

$$\sqrt{3^5} = 3^{\frac{5}{2}}$$

The index of the radical is 2.

Use $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$.

$$\left(\sqrt[3]{25}\right)^2 = 25^{\frac{2}{3}}$$

CHECK YOUR UNDERSTANDING

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a) $8^0 = 1$

b) $8^{1/3} = \sqrt[3]{8} = 2$

c) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

d) $8^{3/3} = 8^1 = 8$

e) $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$

f) $8^{5/3} = (\sqrt[3]{8})^5 = 2^5 = 32$

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